COMPUTER METHODS FOR MATHEMATICAL COMPUTATIONS

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Intended primarily for a course in numerical computing, this text can be used as supplement to a more theoretical text in a numerical analysis course. The programs, subroutines, and exercises for the computer solution of problems involving matrices, integrals, differential equations, spline functions, zeros and extrema of functions, least squares, and Monte Carlo techniques. Intended primarily for a course in numerical computing, this text can be used as supplement to a more theoretical text in a numerical analysis course.